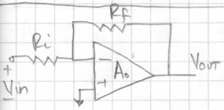
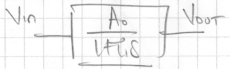
Notes 29 Mar 10

2 Midterms, 1 Final, 2-3 projects

137B → Dynamics
Integrated Circuit Design
Switched Capacitor Design (Z-transforms)



$A_V = -\frac{R_f}{R_i}$ let's say $\boxed{A_V = \frac{A_0}{1+\tau_1 s}}$

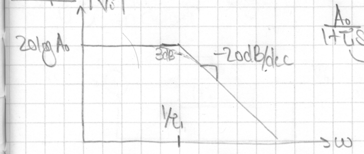


$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A_0}{1+\tau_1 s}$ → pole @ $\frac{1}{\tau_1}$

Any transfer function can be written as

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \dots a_0}{b_m s^m + b_{m-1}s^{m-1} + \dots b_0} = \frac{(s+s_{z1})(s+s_{z2})(\dots)}{(s+s_{p1})(s+s_{p2})(\dots)}$$

Bode plot $\left|\frac{V_o}{V_i}\right|$



$\frac{A_0}{1+\tau_1 s} \rightarrow \frac{A_0}{1+\tau_1 j\omega}$

$20\log A_0 - 20\log(|1+\tau_1 s|)$, $\sqrt{1+\tau_1^2\omega^2} \sim \tau_1\omega$, $\tau_1\omega \gg 1$

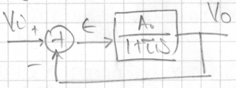
- plot of the Fourier transform not the Laplace transform
- Apply a signal to a stable system and allow transients to settle, then report what happens
- looking at sinusoidal steady state, not concerned with transients
- Must be stable! (LHP only)

when $\omega = 1/\tau_1 \rightarrow \sim \sqrt{2} \sim$ (3dB drop)

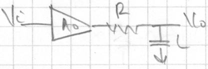
All systems considered are BIBO stable → bounded input bounded output

Phase

- Why is phase important?

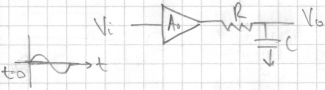


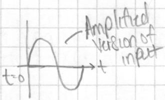
new representation



$\tau_1 = RC$

At DC, input & output has same phase with some time delay between input & output



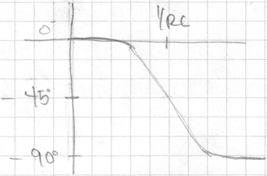


$\omega = \frac{1}{RC}$

↳ both the resistor and the capacitor are the same impedance

↳ so, at 45°

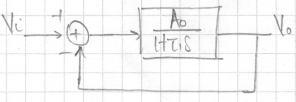
- As system gets faster & faster, phase changes

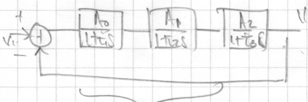


- In response to the need to go faster it will reduce the output amplitude
- −45° implies a lag of 45° with respect to the input

- As frequency increases, it will asymptotically approach −90°
- Every pole adds 90°, signs on the summation change with delay

Example → 3 poles (270°)





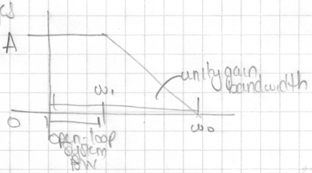
Creates phase inversion

Why do systems have 270°

Single Stage $A_v = 300$ → need multiple stages 30,000

Gain-Bandwidth relationship

$\omega_0 \gg \omega_1$ $\omega_0 = A\omega_1$

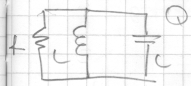


- either have high gain, low bandwidth or low gain, high bandwidth

- Cascade many low gain, high bandwidth systems to get a system that is high gain, high bandwidth

$$A_v = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ ζ = Damping factor

Resonance



- convert all potential energy to kinetic energy & vice versa at a particular point is resonant energy (perfect exchange of energy - resonance)
- Add resistor and energy is lost over time

- If you impose a voltage, cap has no say in dynamics
- Want to impose a current instead to get Bode plot

$$\frac{V_o(s)}{I_n(s)} = \text{impedance of system} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$$

$$= \frac{j\omega RL}{j\omega L + R - \omega^2 LRC} = \frac{j\omega RL}{j\omega L + R - \omega^2 RLC}$$

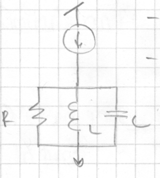
$$= \frac{j\omega L}{j\omega(L/R) + 1 - \omega^2 LC} = \frac{j\omega/C}{j\omega \, 1/RC + 1/LC - \omega^2}$$

$$= \frac{j\omega/C}{1/LC - \omega^2 + j\omega(1/RC)}$$

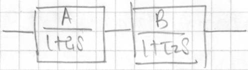
Notes 03-31-10

Second Order Systems

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

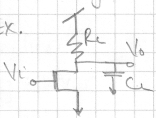


- Generally, don't use inductors except in RF
- What behavior can you get without inductors?



cascade 2 first order systems

Ex.



DC response

$$A_v = -g_m Z_{OUT} = \boxed{-g_m \frac{R_L}{1+sR_LC_L}}$$

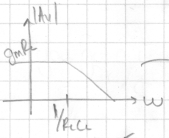
freq. dependent term

$Z_{OUT} = R_L \| r_o \| C_L$

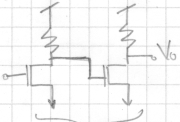
$r_o \gg R_L$

$= R_L \| C_L = \frac{R_L}{1+sC_LR_L}$

$\frac{R_L(1/sC_L)}{R_L + 1/sC_L}$



Add second stage



$\underbrace{\qquad}_{\text{second order system}}$

both poles are real and negative

$$\frac{K}{(s+\alpha)(s+\beta)}$$

$\mathcal{L}^{-1}$

$h(t) = Ce^{-\alpha t} + De^{-\beta t}$

system is stable
always decays

$$\frac{AB}{(1+\tau_1 s)(1+\tau_2 s)}$$

form of eqn is good for partial fractions & getting inverse laplace transforms

$$\frac{\overbrace{\boxed{AB/\tau_1\tau_2}}^{K}}{(\underbrace{1/\tau_1}_{\alpha} + s)(\underbrace{1/\tau_2}_{\beta} + s)}$$

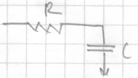
cascading can only give you a second order system that is stable

Step response $u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$

$$H(s) = \frac{K}{(s+\alpha)(s+\beta)} \qquad Y(s) = \frac{H(s)}{s} = \frac{K}{s(s+\alpha)(s+\beta)} \xrightarrow{\mathcal{L}^{-1}}$$

$$y(t) = 1 - (Me^{-\alpha t} + Ne^{-\beta t})$$

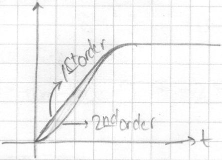
Similar to an RC



$\rightarrow 1 - e^{-t/RC}$

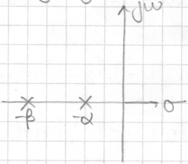
Difference in step response between first & second order systems

In time:



- 2nd order begins as a square function
- 1st order begins linear

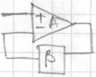
- Can you get a complex system from two real poles?



s-plane $\quad g_m(L) = sL \quad \boxed{g_m(C) = \frac{1}{sC}}$

- make inverse to $\frac{1}{g_m(C)} = sC$
- making a C look like an L

Ideal op amp ex.



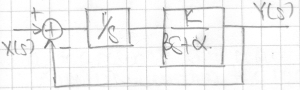
$$= \frac{A}{1+\beta A} \approx \frac{1}{\beta} \qquad i = C\frac{dV}{dt} \qquad V = L\frac{di}{dt}$$

- place integrator in feedback path to get an inductor

$V_c = V_0 \sin \omega_0 t \qquad i_c = V_0 \omega_0 C \cos \omega_0 t$

- Voltage/current out of phase by 90° (Voltage leads)
- Exactly opposite in an inductor

- To make one look like the other, you must shift one by 180°
- Negative feed back accomplishes this



$H(s) = \frac{Y(s)}{X(s)} = \frac{\text{feed forward gain}}{1 + \text{loop gain}}$

feed forward gain $= \frac{1}{s}\left(\frac{K}{\beta s+\alpha}\right)$

loopgain $= \frac{1}{s}\left(\frac{K}{\beta s+\alpha}\right)$

$$H(s) = \frac{\frac{1}{s}\frac{K}{\beta s+\alpha}}{1+\frac{1}{s}\left(\frac{K}{\beta s+\alpha}\right)} = \frac{K}{s(\beta s+\alpha)+K} = \frac{K}{\beta s^2+\alpha s+K}$$

want system to look like $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ ⟶ $\omega_n^2 = K/\beta$
$2\zeta\omega_n = \alpha/\beta$

↑ damping ratio : $\frac{\text{actual}}{\text{critical}}$

roots of denominator

$$\frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4(\omega_n^2)}}{2} = \frac{-2\zeta\omega_n \pm 2\omega_n\sqrt{\zeta^2-1}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

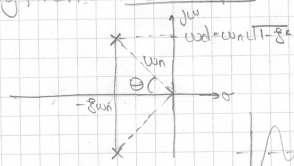
① when $\zeta^2 = 1$ → $-\zeta\omega_n$ (double root → critically damped case)

$\alpha = 2\omega_n\beta$

② when $0 < \zeta^2 < 1$ → (complex roots → underdamped)

③ when $\zeta^2 > 1$ → (real poles → overdamped)

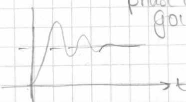
- integrator delays, the lowpass delay more; the closer to 180° delay, the more underdamped

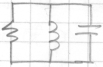


$\theta = \tan^{-1}\left[\frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n}\right]$

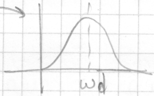
$= \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$

phase delay of sine wave going back





a bandpass filter



a step response will result in amplification of selected sinewaves within the range of the band pass filter

Notes 04-05-2010

Order of poles given order of system

real poles p_1, p_2 → must be complex conjugates of each other if they are the only poles

e^{st} → eigenfunction only scaled by a linear function can only change amplitude and phase

$s=-\sigma \pm j\omega_d \qquad H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}$

$= \frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2} = \frac{\omega_n^2}{s^2+\sigma^2+2\sigma s+\omega_d^2} = \frac{\omega_n^2}{s^2+2\sigma s+\sigma^2+\omega_d^2}$

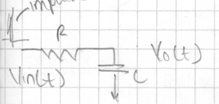
$\sigma = \zeta\omega_n \qquad \omega_n^2=\sigma^2+\omega_d^2 = (\zeta\omega_n)^2+\omega_d^2$

$\omega_n^2(1-\zeta^2)=\omega_d^2 \qquad \omega_d=\omega_n\sqrt{1-\zeta^2}$

$\mathcal{L}\left\{\frac{b}{(s+a)^2+b^2}\right\} = e^{-at}\sin(bt)\,u(t)$

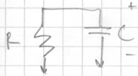
$H(s)= \frac{\omega_n^2}{(s+\sigma)^2+\omega_n^2(\sqrt{1-\zeta^2})} = \frac{\frac{\omega_n^2}{\sqrt{1-\zeta^2}}\sqrt{1-\zeta^2}}{(s+\sigma)^2+\omega_n^2(1-\zeta^2)}$

$= \frac{\omega_n}{\sqrt{1-\zeta^2}}\left[\frac{\omega_n\sqrt{1-\zeta^2}}{(s+\sigma)^2+\omega_n^2(1-\zeta^2)}\right] = \frac{\omega_n}{\sqrt{1-\zeta^2}}\underbrace{\left[\frac{\omega_d}{(s+\sigma)^2+\omega_d^2}\right]}_{\text{impulse response - homogeneous response (unforced)}}$



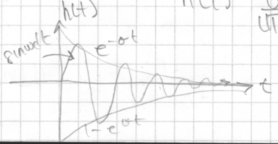
$\frac{C\,dV_o(t)}{dt} = \frac{V_{in}(t)-V_o(t)}{R}$

$\mathcal{L}^{-1}$



zero-input response → $V_{in}(t)=0$

$h(t)=\frac{\omega_n}{\sqrt{1-\zeta^2}}\,e^{-\sigma t}\sin\omega_d t\;u(t)$

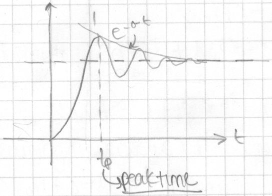


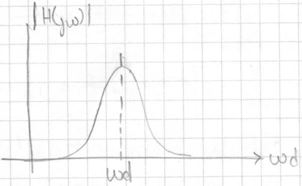
Step Response (edge contains all frequencies)

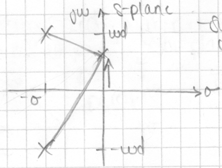
- Integral of the impulse response
- Multiply H(s) by 1/s which is the Laplace transform of the step

$$\mathcal{L}\left\{\frac{a^2+b^2}{s((s+a)^2+b^2)}\right\} \longrightarrow 1-e^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)$$

$$V_{out}(t) = 1 - e^{-\sigma t}\left[\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right] \leftarrow \text{Step response}$$

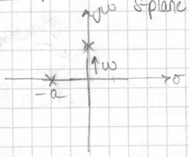


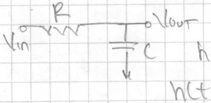




- Sweeping frequencies means you are walking along the $j\omega$-axis
  - measuring resultant vector
  - Shorter vector dominates the response

Example: → includes steady state & transient (most generic form)





$a = 1/RC$

$h(t) = e^{-1/RC}u(t)$

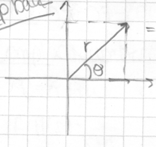
$h(t) = e^{-at}u(t)$

$H(s) = \frac{1}{s+a}$

To find $|H(j\omega)| = \frac{1}{\sqrt{\omega^2+a^2}}$

$H(j\omega) = \frac{1}{j\omega+a}$

Step back



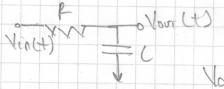
$= re^{j\theta}$ — any complex number represented as a phasor

$= r(\cos\theta + i\sin\theta)$

$r = 1$

$e^{i\theta} = \cos\theta + i\sin\theta$

Back to RC response



$$\frac{C\,dV_{out}(t)}{dt} + \frac{V_{out}(t)}{R} = \frac{V_{in}(t)}{R}$$

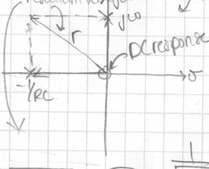
$V_{out}(t) = Ae^{st}$ → b/c it's a real system

substitute: $\frac{C\,dAe^{st}}{dt} + \frac{Ae^{st}}{R} = 0$

$$s(CAe^{st}) + \frac{A}{R}e^{st} = 0 \rightarrow sC + \frac{1}{R} = 0 \rightarrow \boxed{s = \frac{-1}{RC}}$$

- Assuming response is of the form Ae^{st}, but to find sinusoidal $V_{out}(t) = Ae^{j\omega t}$ instead. $\theta = j\omega$ (rotating in s-plane at a certain speed, higher ω = faster rotation)

this plane is $\sigma + j\omega$ not $j\omega$



$\frac{1}{\sqrt{a^2+\omega^2}} = r$

$\sqrt{\omega^2+a^2}$ root of denominator

- as $\omega\uparrow$, response is attenuated, gets smaller

<u>Step back again</u>

Magnitude of $\frac{1}{a+j\omega} \rightarrow re^{j\theta}$

$r = \frac{1}{\sqrt{a^2+\omega^2}}$ $\quad \theta = \tan^{-1}\left(\frac{\omega}{a}\right)$

$$= \underbrace{\frac{1}{\sqrt{a^2+\omega^2}}}\, e^{j\tan^{-1}\left(\frac{\omega}{a}\right)}$$

how did you get this?

$= e^{\sigma + j\omega} = e^{\sigma}(\cos\omega t + j\sin\omega t)$

$H(s) = \frac{1}{s+a}$ if $s = -a \rightarrow H(s) \rightarrow \infty$

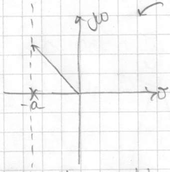
in reality only manipulating $j\omega$, not σ

Notes 04-07-2010

Complex Numbers

$s = \sigma + j\omega$ also can be represented as a vector in the s-plane

if $H(s) = \frac{1}{s+a}$ $\leftarrow |H(s)|$



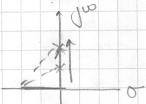
Two vectors $\rightarrow \frac{1}{\underbrace{(s+a)(s+a)}_{\text{multiplication of two vectors}}}$

- In the s-plane is only the variable plane

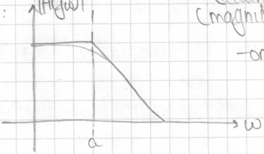
Scaling and rotation
(magnitude) (angle)

Back to first-order system:

$\frac{1}{s+a}$

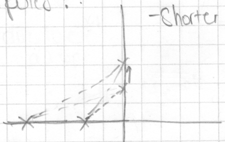


Bode:



- only varying ω (move along jω-axis in s-plane)

Two poles?:



- Shorter of two dominates

Second-order system:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

roots @ $\sigma \pm j\omega_d$

Step response: $\frac{H(s)}{s} = \frac{\omega_n}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$

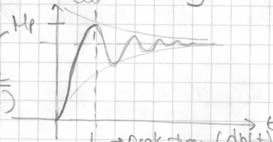
$$\mathcal{L}\left\{\frac{a^2+b^2}{s(s+a)^2+b^2}\right\} = 1 - e^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)$$

$$= \frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2} = \frac{\omega_n^2(\sigma^2+\omega_d^2)}{(\sigma^2+\omega_d^2)[(\sigma+s)^2+\omega_d^2]}$$

$$= \frac{\omega_n^2}{\omega_n^2\zeta^2+\omega_n^2(1-\zeta^2)} = \frac{(\sigma^2+\omega_d)^2}{s((s+\sigma)^2+\omega_d^2)}$$

$$h(t) = 1-e^{-\sigma t}\left[\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right]$$

How does this look?



$$\frac{dh(t)}{dt} = \cancel{\sigma e^{-\sigma t}[\cos\omega_d t} + \frac{\sigma}{\omega_d}\sin\omega_d t] - e^{-\sigma t}[-\omega_d\sin\omega_d t + \cancel{\sigma\cos\omega_d t}]$$

$$\frac{\sigma e^{-\sigma t}}{\omega_d}\sin\omega_d t + e^{\sigma t}\omega_d\sin\omega_d t$$

$$= e^{-\sigma t}\left[\frac{\sigma}{\omega_d}\sin\omega_d t + \omega_d\sin\omega_d t\right] = 0$$

- goes to zero when $\omega_d t \rightarrow \pi$ $\quad \omega_d t = \pi$ $\quad t = t_p$

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

M_p due to $e^{-\sigma t}$ @ t_p $\quad 1-e^{-\sigma t}\left[\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right]$

$$M_p = 1+e^{-\sigma t} = \boxed{1+\exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] = M_p}$$

<u>Stability</u>: BIBO stable (Bounded Input - Bounded Output)

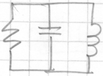
- system with two complex-poles in LHP is BIBO stable
- LHP is always stable due to damping exponential
- if poles on $j\omega$-axis, then you have an oscillation

- Minimum number of poles for oscillation → 3

Simplest oscillator

Ideal - not real (all real systems must have a dissipative term)

but you can never get to $j\omega$ axis

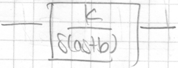


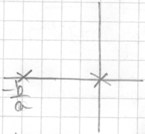
- must have some element to replace energy lost through dissipative term to sustain oscillation

- Once another element is added, the third pole is added which moves other two poles to $j\omega$-axis

$$H(s) = \frac{1}{(s+a)(s+b)}$$

most of time we do this

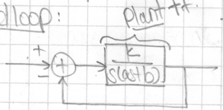




- amplifiers/circuits are typically not used in open loop

- In open loop, signal must be very small or bias will be upset
  for BJT → $\ll \frac{kT}{q}$ for MOS → $\ll V_{ov}$

Closed loop:



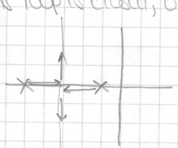
$K \gg 1$

- feedback typically take out large disturbances

- How will the poles of the closed loop system change when open loop gain changes?

Root Locus Method: plot pole locations as K varies

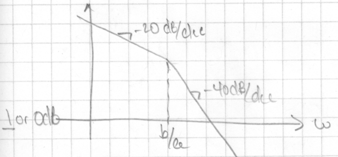
- if loop is open, poles do not move
- As loop is closed, one pole gets faster, one gets slower



Step Response:

- as K increased, poles become complex

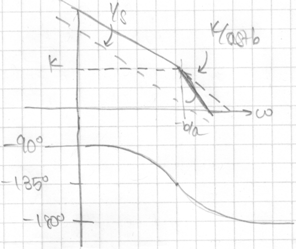
Bode plot method:



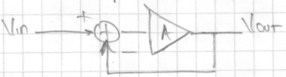
open loop poles

$$\frac{K}{s(as+b)}$$

Summation of $\frac{1}{s}$ & $\frac{K}{as+b}$

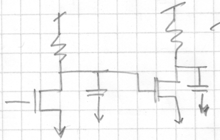


- Anything fed back over 90° will result in ringing
- If you lower K, get less gain at same point so you get a first-order crossover (proportional)
- Called Reduced-gain compensation - loading system to make stable
    - get steady-state error and smaller bandwidth

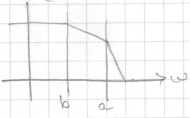


how close you get to Vin depends entirely on A

How can we make an unstable system stable?



two poles $\frac{K}{(s+a)(s+b)}$



If poles at same point

phase is at -90° at the break point